namibia university
of science and technology

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics			
QUALIFICATION CODE:	07BSAM	**LEVEL:**	5
COURSE CODE:	LIA502S	**COURSE CODE:**	LINEAR ALGEBRA 1
SESSION:	JANUARY 2023	**PAPER:**	THEORY
DURATION:	3 HOURS	**MARKS:**	100

SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER:	MR. GS MBOKOMA, DR. N CHERE
MODERATOR:	DR. DSI IIYAMBO

INSTRUCTIONS

1. Attempt all the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in black or blue inked, and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1

1.1 State whether each of the following statements is true or false. *Justify your answer.*

a) If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are any three vectors in $\mathbb{R}^3$, then $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$. [2]

b) $\mathbf{j} \times \mathbf{i} = \mathbf{k}$. [3]

c) If AB and BA are both defined, then A and B are square matrices. [3]

d) If matrix A has a column of all zeros, then so does AB if this product is defined. [3]

e) The expressions $tr(A^TA)$ and $tr(AA^T)$ are defined for every matrix A. [2]

f) The sum of two diagonal matrices of the same size is also a diagonal matrix. [3]

1.2 Given that $\mathbf{u} = \alpha\mathbf{i} + 5\mathbf{j} - \sqrt{7}\mathbf{k}$ and $|\mathbf{u}| = 9$, find the possible values of the scalar α. [4]

1.3 Determine the area of parallelogram whose adjacents sides are $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$. Leave your answer in surd form. [5]

Question 2

2.1 Write down a 4×4 matrix whose ij^{th} entry is given by $a_{ij} = \frac{1}{ij+1}$, and comment on your matrix. [6]

2.2 Let A be a square matrix. State what is meant by each of the following statements.

a) A is symmetric [2]

b) A is orthogonal [2]

c) A is skew-symmetric [2]

2.3 Consider the following matrices.

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{pmatrix}, \quad \text{and } D = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}.$$

a) Given that $C = AB$, determine the element c_{32}. [4]

b) Find $(3A)^T$. [3]

c) Is DB defined? If yes, then find it, and hence calculate $tr(DB)$. [6]

2.4 Suppose A is a square matrix. Check if the matrix $B = 3(A - A^T)$ is skew-symmetric? [5]

Question 3

Consider the matrix $B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -2 & -4 \\ 2 & 3 & -1 \end{pmatrix}$.

a) Is B invertible? If it is, use the Gauss-Jordan Elimination method to find B^{-1}. [12]

b) Find $\det\left(((2B)^{-1})^T\right)$. [4]

Question 4

Use the *Crammer's rule* to solve the following system of linear equations, if it exists.

$$\begin{aligned} 2x - y + 3z &= 2 \\ 3x + y - 2z &= 0 \\ 2x - 2y + z &= 8 \end{aligned}$$

[8]

Question 5

a) Prove that in a vector space, the negative of each vector is unique. [9]

b) Determine whether the following set is a subspace of $\mathbb{R}^3$.

$$S = \{(a,\, b,\, c) \in \mathbb{R}^3 \,|\, a + b + c = 0\}$$

[12]
